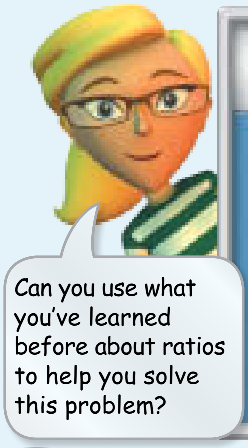
7-2 Similar Polygons

Common Core State Standards

G-SRT.B.5 Use . . . similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

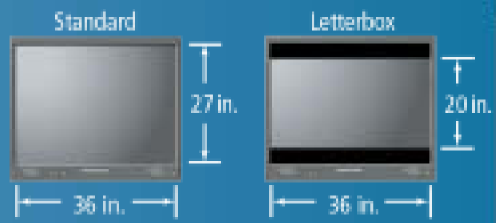
MP 1, MP 3, MP 4, MP 6

Objective To identify and apply similar polygons



SOLVE IT! Getting Ready!

A movie theater screen is in the shape of a rectangle 45 ft wide by 25 ft high. Which of the TV screen formats at the right do you think would show the most complete scene from a movie shown on the theater screen? Explain.



MATHEMATICAL PRACTICES

Similar figures have the same shape but not necessarily the same size. You can abbreviate *is similar to* with the symbol ~.

Essential Understanding You can use ratios and proportions to decide whether two polygons are similar and to find unknown side lengths of similar figures.

Lesson Vocabulary
- similar figures
- similar polygons
- extended proportion
- scale factor
- scale drawing
- scale

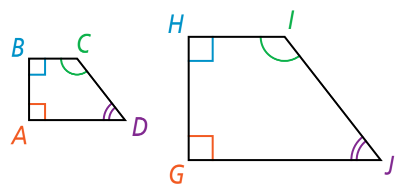
take note **Key Concept** **Similar Polygons**

Define

Two polygons are **similar polygons** if corresponding angles are congruent and if the lengths of corresponding sides are proportional.

Diagram

$ABCD \sim GHIJ$



Symbols

$\angle A \cong \angle G$

$\angle B \cong \angle H$

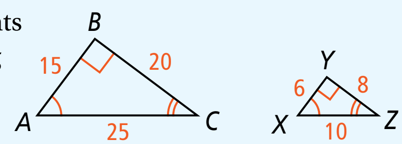
$\angle C \cong \angle I$

$\angle D \cong \angle J$

$\frac{AB}{GH} = \frac{BC}{HI} = \frac{CD}{IJ} = \frac{AD}{GJ}$

You write a similarity statement with corresponding vertices in order, just as you write a congruence statement. When three or more ratios are equal, you can write an **extended proportion.** The proportion $\frac{AB}{GH} = \frac{BC}{HI} = \frac{CD}{IJ} = \frac{AD}{GJ}$ is an extended proportion.

A **scale factor** is the ratio of corresponding linear measurements of two similar figures. The ratio of the lengths of corresponding sides $\overline{BC}$ and $\overline{YZ}$, or more simply stated, the ratio of corresponding sides, is $\frac{BC}{YZ} = \frac{20}{8} = \frac{5}{2}$. So the scale factor of $\triangle ABC$ to $\triangle XYZ$ is $\frac{5}{2}$ or 5 : 2.

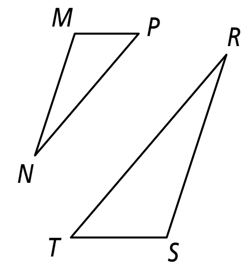


$\triangle ABC \sim \triangle XYZ$

Problem 1 Understanding Similarity

Think
How can you use the similarity statement to write ratios of corresponding sides? Use the order of the sides in the similarity statement. $\overline{MN}$ corresponds to $\overline{SR}$, so $\frac{MN}{SR}$ is a ratio of corresponding sides.

$\triangle MNP \sim \triangle SRT$



A What are the pairs of congruent angles?

$\angle M \cong \angle S$, $\angle N \cong \angle R$, and $\angle P \cong \angle T$

B What is the extended proportion for the ratios of corresponding sides?

$$\frac{MN}{SR} = \frac{NP}{RT} = \frac{MP}{ST}$$

Got It? **1.** $DEFG \sim HJKL$.

a. What are the pairs of congruent angles?

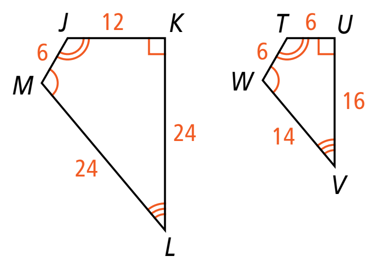
b. What is the extended proportion for the ratios of the lengths of corresponding sides?

Problem 2 Determining Similarity

Think
How do you identify corresponding sides? The included side between a pair of angles of one polygon corresponds to the included side between the corresponding pair of congruent angles of another polygon.

Are the polygons similar? If they are, write a similarity statement and give the scale factor.

A ***JKLM* and *TUVW***



Step 1 Identify pairs of congruent angles.

$\angle J \cong \angle T$, $\angle K \cong \angle U$, $\angle L \cong \angle V$, and $\angle M \cong \angle W$

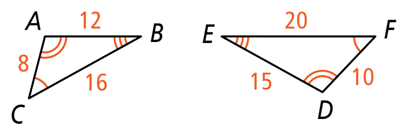
Step 2 Compare the ratios of corresponding sides.

$$\frac{JK}{TU} = \frac{12}{6} = \frac{2}{1} \qquad \frac{KL}{UV} = \frac{24}{16} = \frac{3}{2}$$

$$\frac{LM}{VW} = \frac{24}{14} = \frac{12}{7} \qquad \frac{JM}{TW} = \frac{6}{6} = \frac{1}{1}$$

Corresponding sides are not proportional, so the polygons are not similar.

B **$\triangle ABC$ and $\triangle EFD$**



Step 1 Identify pairs of congruent angles.

$\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$

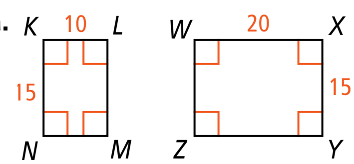
Step 2 Compare the ratios of corresponding sides.

$$\frac{AB}{DE} = \frac{12}{15} = \frac{4}{5} \qquad \frac{BC}{EF} = \frac{16}{20} = \frac{4}{5} \qquad \frac{AC}{DF} = \frac{8}{10} = \frac{4}{5}$$

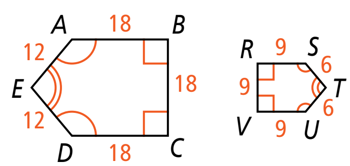
Yes; $\triangle ABC \sim \triangle DEF$ and the scale factor is $\frac{4}{5}$ or $4 : 5$.

Got It? **2.** Are the polygons similar? If they are, write a similarity statement and give the scale factor.

a.



b.

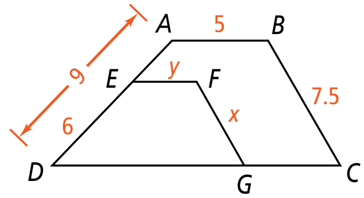


Plan

Can you rely on the diagram alone to set up the proportion?
No, you need to use the similarity statement to identify corresponding sides in order to write ratios that are equal.

Problem 3 Using Similar Polygons

Algebra $ABCD \sim EFGD$. **What is the value of x?**



Ⓐ 4.5 Ⓒ 7.2

Ⓑ 5 Ⓓ 11.25

$\frac{FG}{BC} = \frac{ED}{AD}$ Corresponding sides of similar polygons are proportional.

$\frac{x}{7.5} = \frac{6}{9}$ Substitute.

$9x = 45$ Cross Products Property

$x = 5$ Divide each side by 9.

The value of x is 5. The correct answer is B.

Got It? 3. Use the diagram in Problem 3. What is the value of y?

Think

You can't solve the problem until you know which dimension fills the space first.

Problem 4 Using Similarity

Design **Your class is making a rectangular poster for a rally. The poster's design is 6 in. high by 10 in. wide. The space allowed for the poster is 4 ft high by 8 ft wide. What are the dimensions of the largest poster that will fit in the space?**

Step 1 Determine whether the height or width will fill the space first.

Height: 4 ft = 48 in. Width: 8 ft = 96 in.

48 in. ÷ 6 in. = 8 96 in. ÷ 10 in. = 9.6

The design can be enlarged at most 8 times.

Step 2 The greatest height is 48 in., so find the width.

$\frac{6}{48} = \frac{10}{x}$ Corresponding sides of similar polygons are proportional.

$6x = 480$ Cross Products Property

$x = 80$ Divide each side by 6.

The largest poster is 48 in. by 80 in. or 4 ft by $6\frac{2}{3}$ ft.

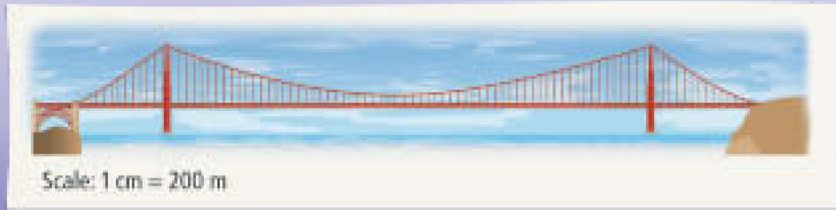
Got It? 4. Use the same poster design in Problem 4. What are the dimensions of the largest complete poster that will fit in a space 3 ft high by 4 ft wide?

In a **scale drawing,** all lengths are proportional to their corresponding actual lengths. The **scale** is the ratio that compares each length in the scale drawing to the actual length. The lengths used in a scale can be in different units. For example, a scale might be written as 1 cm to 50 km, 1 in. = 100 mi, or 1 in. : 10 ft.

You can use proportions to find the actual dimensions represented in a scale drawing.

Problem 5 Using a Scale Drawing STEM

Design **The diagram shows a scale drawing of the Golden Gate Bridge in San Francisco. The distance between the two towers is the main span. What is the actual length of the main span of the bridge?**



The length of the main span in the scale drawing is 6.4 cm. Let s represent the main span of the bridge. Use the scale to set up a proportion.

$$\frac{1}{200} = \frac{6.4}{s} \qquad \frac{\text{length in drawing (cm)}}{\text{actual length (m)}}$$

$$s = 1280 \qquad \text{Cross Products Property}$$

The actual length of the main span of the bridge is 1280 m.

Think
Why is it helpful to use a scale in different units?
1 cm : 200 m in the same units would be 1 cm : 20,000 cm. When solving the problem, $\frac{1}{200}$ is easier to work with than $\frac{1}{20,000}$.

Got It? **5. a.** Use the scale drawing in Problem 5. What is the actual height of the towers above the roadway?

b. **Reasoning** The Space Needle in Seattle is 605 ft tall. A classmate wants to make a scale drawing of the Space Needle on an $8\frac{1}{2}$ in.–by-11 in. sheet of paper. He decides to use the scale 1 in. = 50 ft. Is this a reasonable scale? Explain.

Lesson Check

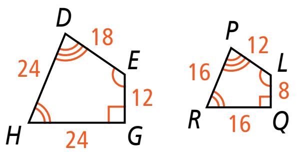
Do you know HOW?

JDRT ~ *WHYX*. Complete each statement.

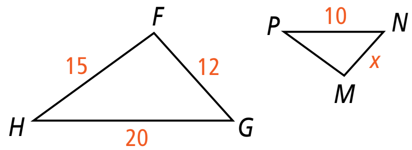
1. $\angle D \cong \underline{\ ?\ }$

2. $\frac{RT}{YX} = \frac{\blacksquare}{WX}$

3. Are the polygons similar? If they are, write a similarity statement and give the scale factor.



4. $\triangle FGH \sim \triangle MNP$. What is the value of x?

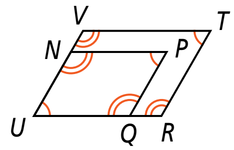


Do you UNDERSTAND?

MATHEMATICAL PRACTICES

5. **Vocabulary** What does the scale on a scale drawing indicate?

6. **Error Analysis** The polygons at the right are similar. Which similarity statement is *not* correct? Explain.

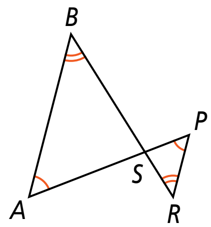


A. *TRUV* ~ *NPQU*

B. *RUVT* ~ *QUNP*

7. **Reasoning** Is similarity reflexive? Transitive? Symmetric? Justify your reasoning.

8. The triangles at the right are similar. What are three similarity statements for the triangles?



Practice and Problem-Solving Exercises

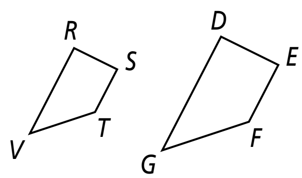
MATHEMATICAL PRACTICES

A Practice

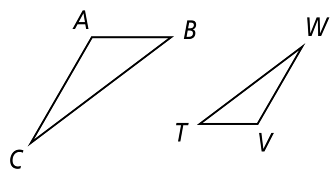
List the pairs of congruent angles and the extended proportion that relates the corresponding sides for the similar polygons.

See Problem 1.

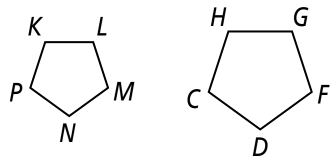
9. *RSTV* ~ *DEFG*



10. $\triangle CAB \sim \triangle WVT$



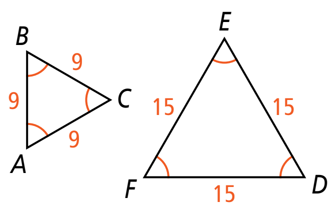
11. *KLMNP* ~ *HGFDC*



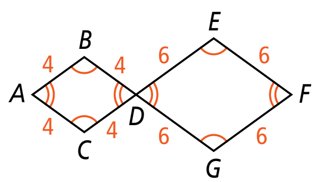
Determine whether the polygons are similar. If so, write a similarity statement and give the scale factor. If not, explain.

See Problem 2.

12.



13.



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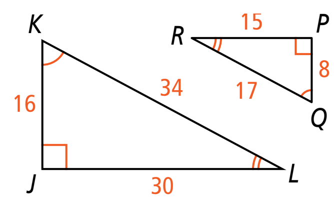
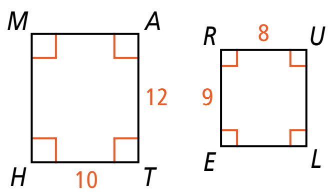
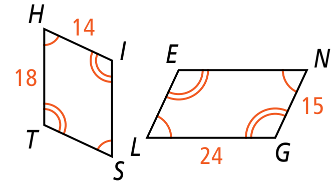
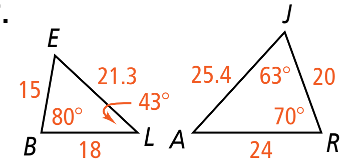
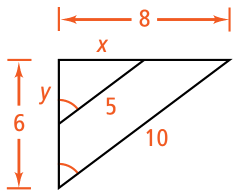


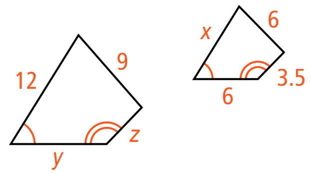
15. 

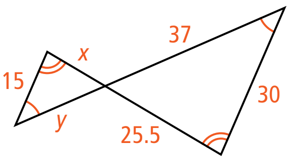
16. 

17. 

Algebra **The polygons are similar. Find the value of each variable.** See Problem 3.

18. 

19. 

20. 

STEM **21. Web Page Design** The space allowed for the mascot on a school's Web page is 120 pixels wide by 90 pixels high. Its digital image is 500 pixels wide by 375 pixels high. What is the largest image of the mascot that will fit on the Web page? See Problem 4.

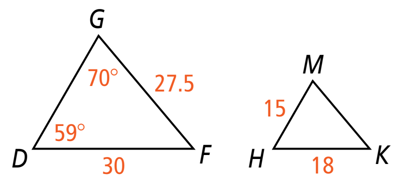
22. Art The design for a mural is 16 in. wide and 9 in. high. What are the dimensions of the largest possible complete mural that can be painted on a wall 24 ft wide by 14 ft high?

STEM **23. Architecture** You want to make a scale drawing of New York City's Empire State Building using the scale 1 in. = 250 ft. If the building is 1250 ft tall, how tall should you make the building in your scale drawing? See Problem 5.

24. Cartography A cartographer is making a map of Pennsylvania. She uses the scale 1 in. = 10 mi. The actual distance between Harrisburg and Philadelphia is about 95 mi. How far apart should she place the two cities on the map?

B Apply

In the diagram below, $\triangle DFG \sim \triangle HKM$. Find each of the following.



25. the scale factor of $\triangle HKM$ to $\triangle DFG$

26. $m\angle K$

27. $\frac{GD}{MH}$

28. MK

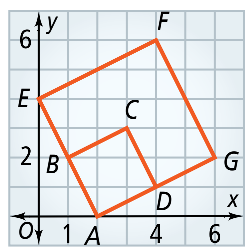
29. GD

30. Flags A company produces a standard-size U.S. flag that is 3 ft by 5 ft. The company also produces a giant-size flag that is similar to the standard-size flag. If the shorter side of the giant-size flag is 36 ft, what is the length of its longer side?

31. a. Coordinate Geometry What are the measures of $\angle A$, $\angle ABC$, $\angle BCD$, $\angle CDA$, $\angle E$, $\angle F$, and $\angle G$? Explain.

b. What are the lengths of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$, $\overline{AE}$, $\overline{EF}$, $\overline{FG}$, and $\overline{AG}$?

c. Is $ABCD$ similar to $AEFG$? Justify your answer.



32. Think About a Plan The Davis family is planning to drive from San Antonio to Houston. About how far will they have to drive?

- How can you find the distance between the two cities on the map?
- What proportion can you set up to solve the problem?

33. Reasoning Two polygons have corresponding side lengths that are proportional. Can you conclude that the polygons are similar? Justify your reasoning.

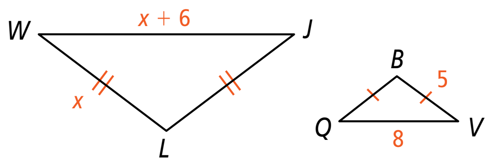
34. Writing Explain why two congruent figures must also be similar. Include scale factor in your explanation.

35. $\triangle JLK$ and $\triangle RTS$ are similar. The scale factor of $\triangle JLK$ to $\triangle RTS$ is 3 : 1. What is the scale factor of $\triangle RTS$ to $\triangle JLK$?

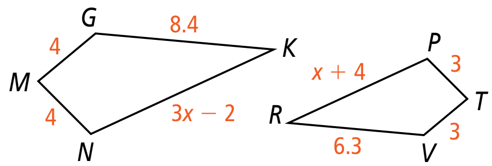
36. Open-Ended Draw and label two different similar quadrilaterals. Write a similarity statement for each and give the scale factor.

Algebra **Find the value of x. Give the scale factor of the polygons.**

37. $\triangle WLJ \sim \triangle QBV$



38. $GKNM \sim VRPT$



Sports **Choose a scale and make a scale drawing of each rectangular playing surface.**

39. A soccer field is 110 yd by 60 yd.

40. A volleyball court is 60 ft by 30 ft.

41. A tennis court is 78 ft by 36 ft.

42. A football field is 360 ft by 160 ft.

Determine whether each statement is *always, sometimes,* or *never* true.

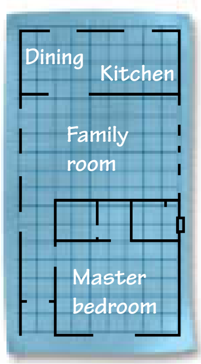
43. Any two regular pentagons are similar.

44. A hexagon and a triangle are similar.

45. A square and a rhombus are similar.

46. Two similar rectangles are congruent.

STEM **47. Architecture** The scale drawing at the right is part of a floor plan for a home. The scale is 1 cm = 10 ft. What are the actual dimensions of the family room?



Challenge

48. The lengths of the sides of a triangle are in the extended ratio 2 : 3 : 4. The perimeter of the triangle is 54 in.

a. The length of the shortest side of a similar triangle is 16 in. What are the lengths of the other two sides of this triangle?

b. Compare the ratio of the perimeters of the two triangles to their scale factor. What do you notice?

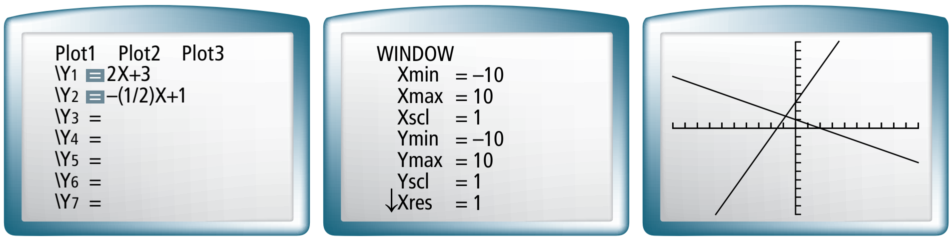
49. In rectangle $BCEG$, $BC : CE = 2 : 3$. In rectangle $LJAW$, $LJ : JA = 2 : 3$. Show that $BCEG \sim LJAW$.

50. Prove the following statement: If $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle GHK$, then $\triangle ABC \sim \triangle GHK$.

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 3

Look back at the information on page 431 about the graph Lillian wants to adjust. The screens on pages 431 are shown again below. In the Apply What You've Learned in Lesson 7-1, you determined the ratio of the width to the height of Lillian's calculator screen.



a. Consider the "viewing rectangle" determined by the values of Xmin, Xmax, Ymin, and Ymax. While the width and height of the calculator screen do not change, the width and height of the viewing rectangle depend on the values entered for Xmin, Xmax, Ymin, and Ymax. What are the width and height of the viewing rectangle for Lillian's graph shown above?

b. Is the viewing rectangle similar to Lillian's rectangular calculator screen? Explain.

c. If the intersecting lines are to be graphed without distortion, what must be true about the viewing rectangle?

d. If Lillian uses Xmin $= -30$ and Xmax $= 30$, will she be able to graph the lines without distortion? Use similarity to explain why or why not.